DESIGN OF FUZZY-MODEL-BASED CONTROLLER FOR TIME-VARYING INPUT-DELAYED TS FUZZY SYSTEMS

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ABSTRACT

This paper addresses a design method of a controller for Takagi-Sugeno (TS) fuzzy systems in the presence of a time-varying input delay. It is commonly believed that the existence of the time delay makes the closed-loop stabilization more difficult. The significance of the addressed control problem is more emphasized in the practical systems such as the virtual laboratory (VL) and the chemical processes. In this paper, a new fuzzy-model-based control methodology is proposed for controlling TS fuzzy systems with time-varying input delay. A new design technique is developed based on the Lyapunov-Razumikhin stability theorem. A sufficient condition for the asymptotic stability of the TS fuzzy systems is formulated in terms of linear matrix inequalities (LMIs). The derived condition can deal with any time-varying input delay within the admissible bound. The effectiveness of the proposed controller design technique is demonstrated by a numerical simulation.

1. INTRODUCTION

Recently, as the communication systems has been more reliable, some attempts have been tried to remotely control via communication networks such as the Internet. Examples include the telerobotic system [17] and the virtual laboratory (VL) [6, 10]. Since the control loops of the remote-control system are closed over communication networks or field buses, time delay phenomena inevitably occur. The stability and performance of the controlled system are definitely dependent on the transmission performance of the communication networks. It is well known that the existence of time delay makes the closed-loop stabilization more difficult. Therefore, it is clear that, as the remote-control system is generally utilized, it will be more and more important to take the delays into account in the analysis and the design of the control systems.

For a few decades, there have been extensive and rigorous studied works related to the delay systems in linear control area. References are actually too many to cite. To name a few, we only mention the studies on the stability of linear system with state delay [2, 5] or with input delay [3, 11] and references therein. The consideration of the time-delay in Takagi-Sugeno (TS) fuzzy-model-based control is also of consequence. However, despite the extensive studies published in the fuzzy-model-based control literature to date [9, 12–14], there are relatively few research results about time delay [7, 8]. However, all previous works did not consider the input delay and are based on the Lyapunov-Krasovskii's stability theorem. Thus it need to take into account some supplementary requirements on the time-derivative of time delay $d(t)$–$\dot{d}(t)$ should be smaller than 1.

Motivated by the above observations, this paper aims at studying the control problem for a class of TS fuzzy systems in the presence of time-varying input delay. The input delays specially often occur in the remote-control system and critically influence on the stability and performance of the closed-loop system. This issue must also be carefully handled in TS fuzzy systems for safety and improved operational performance of the nonlinear remote-control systems such as VL.

This paper proposes some sufficient conditions in terms of linear matrix inequalities (LMIs) and a systematic design procedure for the fuzzy-model-based controller design for a TS fuzzy systems in the presence of the time-varying input delay with admissible bounds. In order to design the controller that stabilize the closed-loop system in the sense of Lyapunov, we utilize the Razumikhin stability theory. By adopting this stability theory, the restriction of $\dot{d}(t)$ is not necessary. This is preferable to the Lyapunov-Krasovskii approach in case of VL, because the traffic rate in the Internet may not satisfy $\dot{d}(t) < 1$. The iterative convex optimization technique is also adopted for the search of the maximal bound of the admissible time delay.

The organization of this paper is as follows: Section

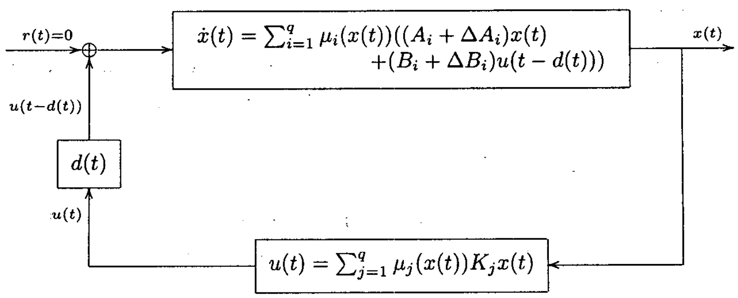

Figure 1: Block diagram of the fuzzy-model-based control system in the presence of time delay and uncertainties.

2 reviews the TS fuzzy system and its basic properties. The main results of this paper are discussed and explained in Section 3. In Section 4, we include a simple example to verify and visualize the theory and method proposed in this paper. To the end, Section 5 concludes this paper with some remarks.

2. TS FUZZY SYSTEMS AND PRELIMINARIES

Consider the TS fuzzy system described by the following fuzzy rules:

Plant Rules

$$R^i: \text{ If } x_1(t) \text{ is } \Gamma_1^i \text{ and } \cdots \text{ and } x_n(t) \text{ is } \Gamma_n^i\,, \quad \text{Then } \dot{x}(t) = A_i x(t) + B_i u(t - d(t))\,, \tag{1}$$

where $\Gamma_j^i (i = 1, \ldots, q, j = 1, \ldots, n.)$ is the fuzzy set, $x(t) \in \mathbb{R}^n$ is the state, $u(t - d(t)) \in \mathbb{R}^m$ is the delayed control input, and $d(t)$ represents the time-lag satisfying $0 \le d(t) \le \tau$.

Using the center-average defuzzification, product inference, and singletone fuzzifier, the global dynamics of this TS fuzzy system (1) is described by

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(x(t))(A_i x(t) + B_i u(t - d(t)))\,, \tag{2}$$

in which

$$\omega_i(x(t)) = \prod_{j=1}^{n} \Gamma_j^i(x_j(t))\,, \quad \mu_i(x(t)) = \frac{\omega_i(x(t))}{\sum_{i=1}^{q} \omega_i(x(t))}\,,$$

and $\Gamma_j^i(x_j(t))$ is the membership value of $x_j(t)$ in Γ_j^i. Some basic properties of $\mu_i(t)$ are:

$$\mu_i(x(t)) \ge 0\,, \quad \sum_{i=1}^{q} \mu_i(x(t)) = 1\,. \tag{3}$$

Throughout this paper, a state feedback TS fuzzy-model-based control law is utilized for the stabilization of the TS fuzzy system (2).

Controller Rules

$$R^i: \text{ If } x_1(t) \text{ is } \Gamma_1^i \text{ and } \cdots \text{ and } x_n(t) \text{ is } \Gamma_n^i\,, \quad \text{Then } u(t) = K_i x(t)\,. \tag{4}$$

The defuzzified output of the controller rules is given by

$$u(t) = \sum_{i=1}^{q} \mu_i(x(t))K_j x(t)\,. \tag{5}$$

The closed-loop system with (2) and (5) of retarded type is represented as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(x(t))\mu_j(x(t - d(t)))(A_i x(t) \\ &\qquad + B_i K_j x(t - d(t)))\,, \\ x(t) &= \phi(t)\,, \quad t \in [-\tau\,, \quad 0]\,, \end{aligned} \tag{6}$$

where $\phi(t)$ is a smooth vector-valued function defined in the Banach space $\mathbb{C}[-\tau\,, \quad 0]$. Figure 1 illustrates the configuration of the control system under consideration.

3. MAIN RESULTS

This section presents some sufficient conditions that guarantee the global asymptotic stability of the controlled TS fuzzy system (6).

Lemma 1 *For given vectors a, b and any symmetric positive definite matrix P of appropriate dimensions, the following inequality holds:*

$$\pm 2a^T b \le \alpha a^T P a + \frac{1}{\alpha} b^T P^{-1} b\,, \tag{7}$$

Remark 1 *In Lemma 1, in case of $-2a^Tb < 0$, the estimated upper bound is not good and may introduce severe conservatism [2]. Nevertheless, the optimization over α can reduce the entered conservatism.*

The main results on the globally asymptotic stability for the closed-loop system (6) is now summarized in the following theorem:

Theorem 1 *If there exist a symmetric positive definite matrix P, and matrices K_j, and positive scalars α_1, α_2 such that the following LMIs are satisfied:*

$$\begin{bmatrix} \frac{1}{\tau}\Upsilon_{ij} + (\alpha_1+\alpha_2)Q & * \\ M_j^T B_i^T & -\frac{1}{2}Q \end{bmatrix} < 0\,, \quad i = 1,2,\ldots,q\,. \tag{8}$$

$$\begin{bmatrix} -\alpha_1 Q & * \\ A_i^T Q & -Q \end{bmatrix} < 0\,, \quad i = 1,2,\ldots,q\,. \tag{9}$$

$$\begin{bmatrix} -\alpha_2 Q & * \\ M_j^T B_i^T & -Q \end{bmatrix} < 0\,, \quad i,j = 1,2,\ldots,q\,. \tag{10}$$

then the TS fuzzy system (2) is robustly globally asymptotically stabilizable by employing controller (5) with the time delay $d(t)$ satisfying

$$0 \le d(t) \le \tau\,,$$

where

$$\Upsilon_{ij} = QA_i^T + A_iQ + M_j^T B_i^T + B_iM_j$$

with $Q = P^{-1}, M_j = K_jP^{-1}$, and $*$ denotes the transposed elements in the symmetric positions.

Proof: Choose a Lyapunov functional candidate as

$$V(x(t)) = x(t)^T Px(t)\,. \tag{11}$$

Clearly, $V(x(t))$ is positive definite and radially unbounded. The time derivative of $V(x(t))$ of (6) is given by

$$\begin{aligned} \dot V(x(t)) &= \dot x(t)^T Px(t) + x(t)^T P\dot x(t) \\ &= \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(x(t))\mu_j(x(t-d(t)))((A_ix(t) \\ &\quad + B_iK_jx(t-d(t)))^T Px(t) \\ &\quad + x(t)^T P(A_ix(t) + B_iK_jx(t-d(t))))\,. \end{aligned} \tag{12}$$

Note that

$$\begin{aligned} x(t-d(t)) &= x(t) - \int_{t-d(t)}^{t} \dot x(\theta)d\theta \\ &= x(t) - \int_{t-d(t)}^{t} \sum_{k=1}^{q}\sum_{l=1}^{q} \mu_k(\theta)\mu_l(\theta-d(\theta)) \\ &\quad \times (A_kx(\theta) + B_kK_lx(\theta+d(\theta))d\theta\,. \end{aligned} \tag{13}$$

Then, substituting (13) into (12) results in

$$\begin{aligned} &\dot V(x(t)) \\ &= \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(x(t))\mu_jx(t-d(t)) \\ &\times (x(t)^T(A_i^TP + PA_i + K_j^TB_i^TP + PB_iK_j)x(t) \\ &-2\int_{t-d(t)}^{t} x(t)^T PB_iK_j\dot x(\theta)d\theta \Big) \\ &= \sum_{i=1}^{q} \mu_i(x(t))\mu_jx(t-d(t)) \\ &\times (x(t)^T(A_i^TP + PA_i + K_j^TB_i^TP + PB_iK_j)x(t) \\ &-2\int_{t-d(t)}^{t} x(t)^T PB_iK_j \Big(\sum_{k=1}^{q}\sum_{l=1}^{q} \mu_k(x(\theta))\mu_l(x(\theta)) \\ &\times ((A_kx(\theta) + B_kK_lx(\theta-d(\theta))d\theta)\,. \end{aligned} \tag{14}$$

Applying Lemma 1 to (14) and from the property of the firing strength (3), (14) is less than

$$\begin{aligned} &\sum_{i=1}^{q}\sum_{j=1}^{q}\sum_{k=1}^{q}\sum_{l=1}^{q} \mu_i(x(t))\mu_jx(t-d(t)) \\ &\times (x(t)^T(A_i^TP + PA_i + K_j^TB_i^TP + PB_iK_j)x(t) \\ &\quad + \int_{t-d(t)}^{t} \frac{1}{\alpha_1}x(t)^T PB_iK_jA_kP^{-1}A_k^TK_j^TB_i^TPx(t) \\ &\quad + \alpha_1x(\theta)^TPx(\theta) \\ &\quad + \frac{1}{\alpha_2}x(t)^TB_iK_jB_kK_lP^{-1}K_l^TB_k^TK_j^TB_i^Tx(t) \\ &\quad +\alpha_2x(\theta-d(\theta))^TPx(\theta-d(\theta))d\theta)\,. \end{aligned} \tag{15}$$

Keeping the Razumikhin stability theorem in mind [1], and assuming that for any real number $\delta > 1$, we have

$$V(x(\theta)) < \delta V(x(t))\,, \quad \forall\theta \in [t-2\tau\,,\ \ t]\,. \tag{16}$$

Using Schur complement, the following inequalities directly follows from LMIs (9) and (10).

$$A_kP^{-1}A_k^T \le \alpha_1P^{-1}\,, \quad B_kK_lP^{-1}K_l^TB_k^T \le \alpha_2P^{-1}\,,$$
$$k = 1,2,\ldots,q.,\quad l = 1,2\ldots,q.$$

Then, it is not difficult to understand that the right side of the above inequality (15) is less than

$$\begin{aligned} &\sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(x(t))\mu_jx(t-d(t)) \\ &\times (x(t)^T(A_i^TP + PA_i + K_j^TB_i^TP + PB_iK_j)x(t) \\ &+ 2d(t)x(t)^TPB_iK_jP^{-1}K_j^TB_i^TPx(t) \\ &+ d(t)\delta(\alpha_1+\alpha_2)x(t)^TPx(t))\,. \end{aligned} \tag{17}$$

From the observation that the right side of (17) is monotonically increasing with regards to the time delay $d(t)$, then, if the following nonlinear matrix inequalities hold for $x(t)$ and for all $t \geq 0$, except at $x(t) = 0$,

$$\begin{aligned} & A_i^T P + PA_i + K_j^T B_i^T P + PB_i K_j \\ & + 2\tau PB_i K_j P^{-1} K_j^T B_i^T P + \tau\delta(\alpha_1 + \alpha_2)P + < 0, \\ & \qquad i = 1, 2, \ldots, q, \qquad (18) \end{aligned}$$

then the controlled system (6) is globally asymptotically stable in the sense of Lyapunov with admissible bound τ. Moreover, from the continuity of the eigenvalues (18) with respect to δ, there exists a $\delta > 1$ sufficiently small such that (18) with $\delta \doteq 1$ still hold.

With some matrix manipulations, we can show that (8), immediately implies the global asymptotic stability of the controlled system (6). This completes the proof.

■

In order to find the maximum delay τ, the following convex optimization algorithm is proposed.

Step 1: Find a positive definite matrix Q and matrices M_j such that the following LMIs are satisfied:

$$QA_i^T + A_iQ + M_j^T B_i^T + B_i M_j < 0, \quad i, j = 1, 2, \ldots, q.$$

Step 2: For Q given in the previous step, find α_1, α_2 and M_j such that the following generalized eigenvalue problem (GEVP) $\mathcal{P}(\tau)$ has solutions,

$$\begin{aligned} \mathcal{P}(\tau) \quad & \max_{M_j, \alpha_1, \alpha_2} \tau \quad \text{subject to} \\ & \begin{bmatrix} \frac{1}{\tau}\Upsilon_{ij} + (\alpha_1 + \alpha_2)Q & * \\ M_j^T B_i^T & -\frac{1}{2}Q \end{bmatrix} < 0, \\ & \begin{bmatrix} -\alpha_1 Q & * \\ A_i^T Q & -Q \end{bmatrix} < 0, \\ & \begin{bmatrix} -\alpha_2 Q & * \\ M_j^T B_i^T & -Q \end{bmatrix} < 0, \quad i, j = 1, 2, \ldots, q. \end{aligned}$$

Step 3: For M_j and α_1, α_2 given in the previous step, find Q such that $\mathcal{P}(\tau)$ has solutions.

Step 4: Return to Step 2 until the convergence of τ is obtained with a desired precision.

4. AN EXAMPLE

In this section, a numerical example is presented for illustrating the controller design technique proposed in Section 3.. Consider the following TS fuzzy systems.

Plant Rules

R^1: If $x_1(t)$ is about Γ_1,
THEN $\dot{x}(t) = A_1 x(t) + B_1 u(t - d(t))$,

R^2: If $x_1(t)$ is about Γ_2,
THEN $\dot{x}(t) = A_2 x(t) + B_2 u(t - d(t))$,

where

$$A_1 = \begin{bmatrix} -0.5 & 0.1 \\ 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 0.1 \\ 1 & 0 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The membership functions are

$$\Gamma_1(x_1(t)) = \begin{cases} 0, & x_1(t) < -\Omega, \\ 1 - \frac{x_1^2(t)}{\Omega^2}, & -\Omega \leq x_1(t) \leq \Omega, \\ 0, & \Omega < x_1(t), \end{cases}$$
$$\Gamma_2(x_1(t)) = 1 - \Gamma_1(x_1(t)),$$

where $\Omega = 0.8165$. From Theorem 1 and the proposed iterative optimization method, we get

$$P = \begin{bmatrix} 0.0171 & 0.0103 \\ 0.0103 & 0.0098 \end{bmatrix}, \quad \alpha_1 = 1.3133, \quad \alpha_2 = 0.6620,$$
$$K_1 = \begin{bmatrix} -0.7154 & -0.2981 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -0.7155 & -0.2980 \end{bmatrix},$$

and $\tau = 0.2639$, which means that the designed TS fuzzy-model-based controller can robustly stabilize the TS fuzzy system against any time-varying input delay $d(t) \leq \tau = 0.2639$.

The initial value is $x(0) = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$. During the simulation process, the time-varying delay in the control law is assumed as

$$d(t) = \frac{\tau}{2}(\sin(100t) + 1),$$

where the assumed time delay does not exceed the upper bound, $\tau = 0.2639$ and its maximal time-derivative is $50\tau = 13.195 > 1$. To the authors' knowledge, the stability analysis based on the Lyapunov-Krasovskii functional approach demands the restriction of the time-derivative of $d(t)$, i.e. $\dot{d}(t) < 1$. On the other hand, the proposed method do not need it. It also implies that the Lyapunov-Razumikhin stability theory is much suitable for the remote-control system based

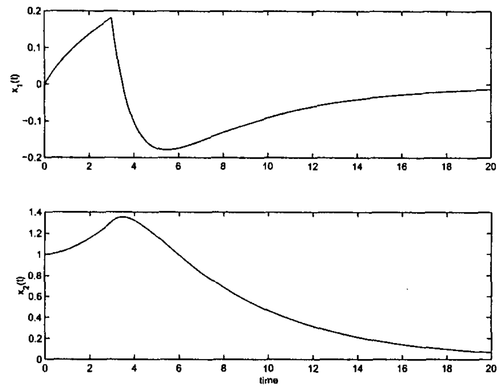

Figure 2: The controlled response of the time-varying input-delayed TS fuzzy system.

on network channels such as the Internet, because the time delay due to the traffic jam over the Internet may randomly varied.

The simulation result is shown in Fig. 2. For the purpose of comparison, the control input is activated at $t = 3$ sec.. Before the control input is activated, the trajectories of the system do not go to the equilibrium of the system. However, after $t = 3$ sec., the trajectories of the controlled system are quickly guided to the origin. From the simulation result, one can see that the designed controller can stabilize the TS fuzzy system with the input delay.

5. CONCLUSIONS

In this paper, the problem of stabilization of the TS fuzzy system with time-varying input delay has been addressed. In order to design the fuzzy-model-based controller, the Lyapunov-Razumikhin stability theorem has been applied. The sufficient condition for the stabilization of the closed-loop system has been given in terms of the linear matrix inequalities (LMIs). The maximal bound of the input delay preserving the asymptotic stability has been found by using the iterative convex optimization technique. From the numerical example has shown us the potential of the proposed method for the industrial applications with the delay phenomena.

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